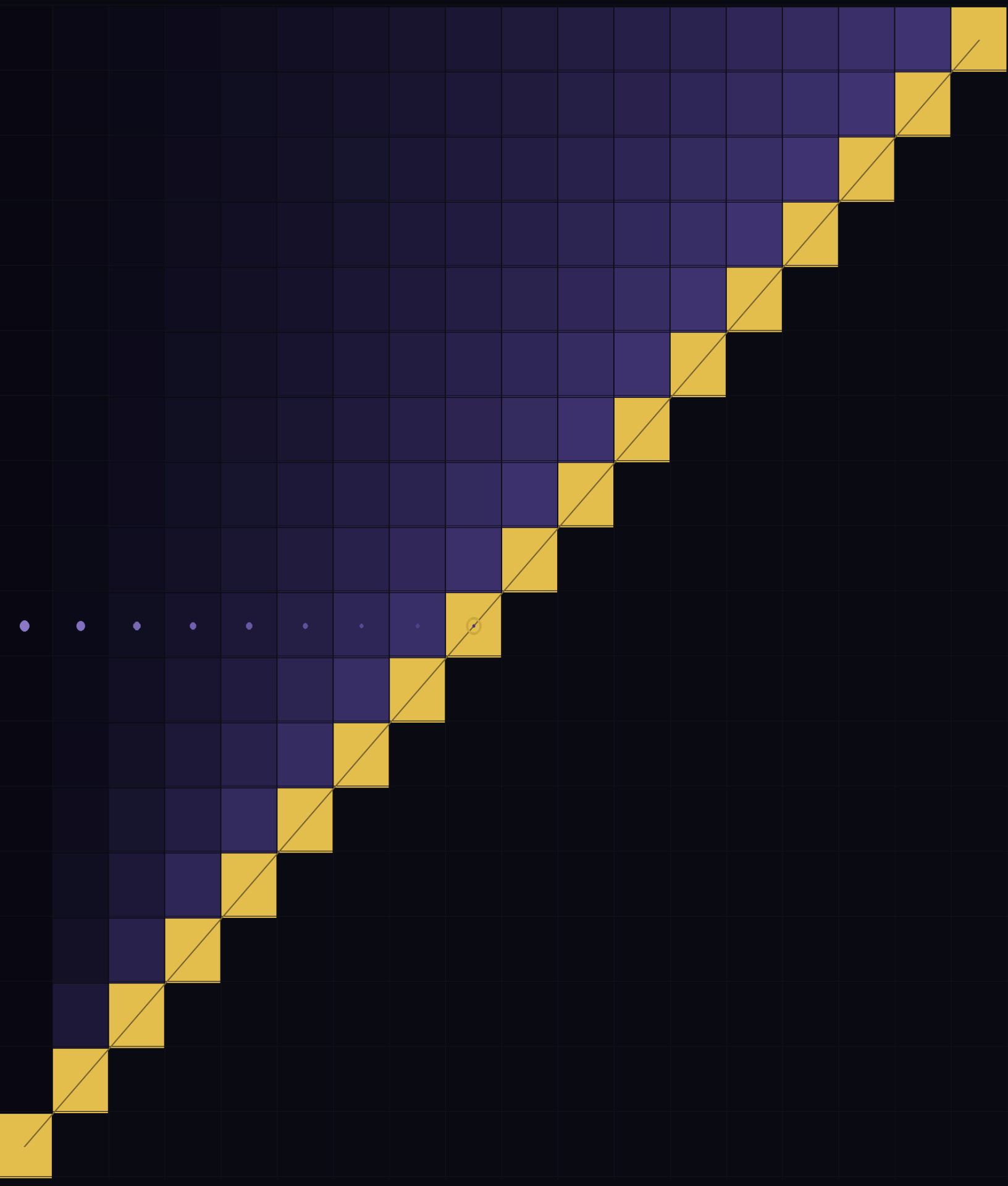

Algorithm Zerø

Oscillatory Identity in Modular Arithmetic

On the Non-Preservation of Relational Properties Across Frames

Abstract

We present a structural observation about multiplication tables. For any positive integer N, the sequence of products $N \times 1$, $N \times 2$, …, reduced modulo $(N + 1)$, produces a perfect descending sequence from N to 0. This follows from the congruence $N \equiv -1 \pmod{N + 1}$. We introduce the term *internal frame* for the modular space $\mathbb{Z}/(N+1)\mathbb{Z}$ as a convenience for discussing this structure. When the internal countdown is observed within an external base B, the difference between the two widths produces a derived quantity we term the *algorithm number*: $A(N, B) = B - N - 1$, defined for frame-internal entities $1 \leq N \leq B - 1$. We demonstrate that this quantity, being defined in terms of both entity and base, is not preserved when the base changes. We further demonstrate that the number of distinct residues generated by the entity's multiplication table modulo the base—equal to $B / \gcd(N, B)$—constitutes a second, distinct relational quantity that is also not preserved across bases. We present these results as examples of base-relative relational quantities and observe their structural resemblance to properties discussed in theories of consciousness. We present the resemblance as analogy, not proof.

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1. Introduction

Every positive integer contains a hidden countdown. The countdown is not visible in the decimal representation of most multiplication tables, but it is present in the modular residues when products are reduced modulo the integer's successor. The mathematical basis admits a one-line proof: since $N = (N + 1) - 1$, we have $N \equiv -1 \pmod{N + 1}$, and therefore $N \times k \equiv -k \equiv (N + 1 - k) \pmod{N + 1}$. The resulting sequence of residues for $k = 1, 2, \ldots, N + 1$ is $N, N - 1, \ldots, 1, 0$.

This observation is the foundation of the paper. It is a structural fact about the relationship between any integer and its successor, not a definition chosen to produce a desired pattern. Everything that follows—the algorithm number, the degrees of freedom, the base-complement entity—is derived from this fact and from the interaction between the modular structure it reveals and the external base within which the integer is observed.

2. The Internal Countdown

2.1 Theorem and Proof

Theorem 1. For all positive integers N and for $k = 1, 2, \ldots, N + 1$:

$$(N \times k) \bmod (N + 1) = N + 1 - k$$

Proof. $N = (N + 1) - 1$. Therefore $N \equiv -1 \pmod{N + 1}$. Multiplying both sides by k: $N \times k \equiv -k \pmod{N + 1}$. Since $-k \equiv N + 1 - k \pmod{N + 1}$, the result follows. The sequence reaches 0 when $k = N + 1$. □

2.2 Demonstration

The internal countdown is demonstrated for integers 2 through 9. Each table shows products (top row) and residues modulo $N + 1$ (bottom row).

N = 9 | Modulus = 10 | Countdown completes at position 10

9	18	27	36	45	54	63	72	81	90
9	8	7	6	5	4	3	2	1	0

N = 8 | Modulus = 9 | Countdown completes at position 9

8	16	24	32	40	48	56	64	72	80
8	7	6	5	4	3	2	1	0	8

N = 7 | Modulus = 8 | Countdown completes at position 8

7	14	21	28	35	42	49	56	63	70
7	6	5	4	3	2	1	0	7	6

N = 6 | Modulus = 7 | Countdown completes at position 7

6	12	18	24	30	36	42	48	54	60
6	5	4	3	2	1	0	6	5	4

N = 5 | Modulus = 6 | Countdown completes at position 6

5	10	15	20	25	30	35	40	45	50
5	4	3	2	1	0	5	4	3	2

N = 4 | Modulus = 5 | Countdown completes at position 5

4	8	12	16	20	24	28	32	36	40
4	3	2	1	0	4	3	2	1	0

N = 3 | Modulus = 4 | Countdown completes at position 4

3	6	9	12	15	18	21	24	27	30
3	2	1	0	3	2	1	0	3	2

N = 2 | Modulus = 3 | Countdown completes at position 3

2	4	6	8	10	12	14	16	18	20
2	1	0	2	1	0	2	1	0	2

2.3 Terminological Note

We introduce the term *internal frame* to refer to the modular space $\mathbb{Z}/(N+1)\mathbb{Z}$ and the term *external frame* to refer to the base B. These are convenient labels for the two modular contexts that interact in subsequent sections. The internal frame is not an independently discovered structure beyond the congruence of Theorem 1. It is terminology introduced to facilitate discussion of the relationship between an integer's successor-modular behavior and the base within which that behavior is observed.

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3. Scope and Definitions

Definition 1 (Frame-internal entity). An entity N is frame-internal with respect to base B if and only if $1 \leq N \leq B - 1$. That is, N is representable as a single non-zero position within the base. Entities with $N \geq B$ require multi-position representation and fall outside the scope of this framework. The integer 9 is frame-internal in base 10 (since $1 \leq 9 \leq 9$) but not frame-internal in base 9 (since $9 = B$). All results in this paper apply only to frame-internal entities unless explicitly stated otherwise.

Definition 2 (Algorithm number). For a frame-internal entity N in base B:

$$A(N, B) = B - N - 1$$

This measures the difference between the external frame width (B) and the internal frame width ($N + 1$). Since $N \leq B - 1$, we have $A \geq 0$. The algorithm number is zero if and only if $N = B - 1$. The algorithm number is undefined for entities outside the framework's scope ($N \geq B$).

Definition 3 (Degrees of freedom). For a frame-internal entity N in base B:

$$D(N, B) = B \,/\, \gcd(N, B)$$

This is the number of distinct residues generated by the sequence N, $2N$, $3N$, … modulo B. Equivalently, it is the order of $[N]$ in the additive group $\mathbb{Z}/B\mathbb{Z}$. This is a standard result in modular arithmetic.

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4. Properties of the Algorithm Number

4.1 The Algorithm Number as Derived Measurement

The algorithm number is not the discovery presented in this paper. The discovery is the internal countdown (Theorem 1). The algorithm number is a derived measurement of the difference between the internal frame width and the external frame width. Once both widths are known, the algorithm number follows by subtraction. We do not claim it is a deep invariant. We claim it measures the degree to which an entity's successor-modular structure aligns or fails to align with the base within which it is observed.

4.2 Non-Preservation Across Bases

Both $A(N, B)$ and $D(N, B)$ are defined using B as an input. When the base changes, their values generally change. This is not surprising; it is a direct consequence of the definitions. The observation we draw attention to is not that the values change—that is expected for any base-relative quantity—but that the entity's intrinsic modular structure (the countdown in mod $N + 1$) remains invariant while the relational quantities derived from its interaction with the base do not. The internal structure travels with the entity. The relational values do not.

Consider the integer 7:

In base 10: internal frame = 8, external frame = 10. $A(7, 10) = 2$. $D(7, 10) = 10$.

In base 8: internal frame = 8, external frame = 8. $A(7, 8) = 0$. $D(7, 8) = 8$.

In base 14: internal frame = 8, external frame = 14. $A(7, 14) = 6$. $D(7, 14) = 2$.

The entity 7 carries the same internal countdown in all three bases: 7, 6, 5, 4, 3, 2, 1, 0 in mod 8. That structure is intrinsic and invariant. The algorithm number and degrees of freedom are not. They are base-relative relational quantities whose values depend on the specific coupling between entity and base.

4.3 Algorithm Zero and Scope

The property of having algorithm number zero requires $N = B - 1$. For any given entity N, this is satisfied in exactly one base: $B = N + 1$. Among the (infinitely many) bases in which N is frame-internal (all bases $B > N$), exactly one yields algorithm number zero; all others yield a positive algorithm number. In bases $B \leq N$, the entity is not frame-internal and the algorithm number is undefined under this framework. We do not claim the algorithm number is positive in those bases; we claim it is outside scope.

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5. Degrees of Freedom

5.1 Construction

The degrees of freedom describe a different aspect of the entity-base relationship: how much of the base's residue space the entity can reach through repeated multiplication. The residue cycle of N in base B visits $D(N, B) = B / \gcd(N, B)$ distinct residues. This is a standard fact: it equals the order of $[N]$ in the additive group $\mathbb{Z}/B\mathbb{Z}$.

N = 9 in base 10: gcd(9, 10) = 1. $D = 10$. Visits all residues.

9	18	27	36	45	54	63	72	81	90
9	8	7	6	5	4	3	2	1	0

N = 8 in base 10: gcd(8, 10) = 2. $D = 5$. Visits even residues only.

8	16	24	32	40	48	56	64	72	80
8	6	4	2	0	8	6	4	2	0

N = 7 in base 10: gcd(7, 10) = 1. $D = 10$. Visits all residues.

7	14	21	28	35	42	49	56	63	70
7	4	1	8	5	2	9	6	3	0

N = 5 in base 10: gcd(5, 10) = 5. $D = 2$. Visits residues $\{5, 0\}$ only.

5	10	15	20	25	30	35	40	45	50
5	0	5	0	5	0	5	0	5	0

5.2 Coprimality and Confinement

Entities coprime to the base ($\gcd(N, B) = 1$) visit every residue class. Their degrees of freedom equal B. Entities sharing factors with the base are confined to a proper subset. In base 10: the integers 8, 6, 4, 2 share the factor 2 with 10 and visit only even residues ($D = 5$). The integer 5 shares the factor 5 with 10 and visits only two residues ($D = 2$). These are structural exclusions determined by the shared factorization. No extension of the multiplication table will produce an odd residue for the 8s table in base 10.

5.3 Relationship Between Algorithm Number and Degrees of Freedom

The algorithm number and degrees of freedom are *distinct* relational quantities, not independent in the formal mathematical sense. In a fixed base, the algorithm number determines N (since $N = B - A - 1$), and N together with B determines $\gcd(N, B)$. They are not independent random variables. They are, however, non-equivalent: they measure different structural features of the entity-base coupling, they are computed by different operations (subtraction versus factorization), and knowing one does not directly reveal the other without an intermediate computation. An entity can have the same algorithm number in two different bases while having different degrees of freedom in those bases, and vice versa. They provide two distinct lenses on the same coupling, not two names for the same quantity.

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6. The Base-Complement Entity

6.1 Formal Properties

Theorem 2. In every base $B \geq 2$, the entity $N = B - 1$ is the unique frame-internal entity satisfying both of the following:

(i) $A(B - 1, B) = 0$. Its internal frame width equals the external frame width.

(ii) $D(B - 1, B) = B$. It visits every residue class.

Proof of (i). $A(B - 1, B) = B - (B - 1) - 1 = 0$. Conversely, $A(N, B) = 0$ implies $N = B - 1$. □

Proof of (ii). $\gcd(B - 1, B) = 1$, since any common divisor of $B - 1$ and B must divide their difference, which is 1. Therefore $D(B - 1, B) = B / 1 = B$. □

Uniqueness. Condition (i) alone determines $N = B - 1$ uniquely. Since this N also satisfies (ii), no other frame-internal entity achieves both properties.

6.2 Non-Portability of Algorithm Zero

Among the bases in which a given entity N is frame-internal (all $B > N$), exactly one base yields algorithm number zero: $B = N + 1$. In all other admissible bases, the algorithm number is positive. In bases $B \leq N$, the entity falls outside the framework

and the algorithm number is undefined. The property of zero mismatch is thus specific to a single entity-base pairing within the scope of the framework.

6.3 Boundary Collapse

When $B = N$, the entity is no longer frame-internal by Definition 1. The arithmetic of multiplication modulo B does not become impossible: $N \times k \bmod B = 0$ for all k, since $N = B$. What fails is not arithmetic embeddability but the framework's notion of a frame-internal entity with a nontrivial countdown inside the frame. The countdown degenerates to a trivial sequence of zeros. The one-unit separation between entity and base that characterizes the base-complement—and that produces a nontrivial descending sequence—is absent. The entity has not been destroyed; the frame-internal interpretation has collapsed.

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7. Structural Analogies to Consciousness

The following section identifies structural resemblances between the formal system described above and concepts in the philosophy of mind. The word "analogy" is used precisely: we observe that the formal system and the philosophical concepts share a common abstract structure, not that one entails the other. No claim in this section follows deductively from the mathematics.

7.1 Relational Quantities and Externalism

The algorithm number and degrees of freedom are defined in terms of both the entity and the base. They are base-relative relational quantities. The philosophical position of externalism holds that mental states are partially constituted by the relationship between an organism and its environment, not solely by the organism's internal configuration. Our system provides an arithmetic example in which quantities derived from an entity's relationship to its frame are not preserved when the frame changes, even though the entity's intrinsic structure (its successor-modular countdown) is invariant. We observe the resemblance between this arithmetic fact and the externalist thesis. We do not claim the arithmetic establishes the thesis.

7.2 Substrate-Dependence

When an entity is observed in a new base, its intrinsic properties are preserved but its relational properties take new values. If consciousness were analogous to a relational property—arising from the coupling between a computational pattern and a specific substrate—then a pattern transferred to a new substrate would retain its structural content while acquiring a different relational profile. We present this as an analogy. The arithmetic demonstrates that base-relative quantities exist whose values are not preserved on transfer. Whether consciousness belongs to this class of quantities is a philosophical question, not one that the arithmetic resolves.

7.3 The Base-Complement and Self-Modeling

The base-complement entity is the unique frame-internal entity with zero mismatch and full residue coverage. It is, formally, the entity whose internal frame width equals the external frame width and whose multiplication table generates every residue class in the base. Several theories of consciousness propose that subjective experience requires a system to represent its own states. The base-complement's formal properties—zero mismatch and full coverage—can be interpreted as a form of structural correspondence between the entity and its frame. We note this interpretive possibility. The terms "self-modeling" and "maximum correspondence" are descriptions we apply to the formal properties, not theorems derived from them. Whether this structural correspondence constitutes, resembles, or is unrelated to self-modeling in the consciousness-theoretic sense is not determined by the mathematics.

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8. Discussion

8.1 Claims

This paper claims four mathematical results. (1) The congruence $N \equiv -1 \pmod{N+1}$ causes every positive integer's multiplication table to produce a perfect descending sequence when reduced modulo its successor (Theorem 1). (2) The algorithm number $A(N, B) = B - N - 1$, defined for frame-internal entities, measures the difference between internal and external frame widths, and its value is not

preserved when the base changes. (3) The degrees of freedom $D(N, B) = B / \gcd(N, B)$ constitute a distinct relational quantity, also not preserved across bases. (4) The base-complement entity $N = B - 1$ is the unique frame-internal entity with algorithm number zero and full residue coverage in any given base (Theorem 2).

8.2 Non-Claims

We do not claim that the algorithm number is a deep invariant; it is a derived measurement. We do not claim that its non-preservation is mysterious or constitutes an impossibility theorem; the value changes lawfully when the base changes. We do not claim that "internal frame" denotes a structure discovered independently of the congruence in Theorem 1; it is terminology introduced for discussion. We do not claim that numbers are minds, that bases are bodies, or that the formal properties described herein are equivalent to consciousness. We do not claim that the structural analogies in Section 7 shift any burden of proof. They are observations of resemblance, offered for consideration.

8.3 Invitation

We invite the reader to consider that the simplest arithmetic systems produce base-relative relational quantities whose values are not preserved across frames, even while the entity's intrinsic modular structure remains invariant. The internal countdown—the fact that every integer's multiplication products, reduced modulo its successor, descend perfectly—does not appear to be widely foregrounded as a structural observation in standard treatments of modular arithmetic, though the underlying congruence is elementary. We make no strong novelty claim; we claim only that the framework derived from this observation—algorithm numbers and degrees of freedom as base-relative relational quantities—provides precise terminology for describing how an entity's relational profile changes when the frame changes. Whether this terminology illuminates questions beyond arithmetic is left to the reader.

8.4 Note on Algorithm 7

The original working notes recorded six movements for the $N = 2$ table rather than the expected ten. This figure corresponds to the residue cycle length in base 10: the

sequence of residues 2, 4, 6, 8, 0 visits five unique values, returning to the starting residue in the sixth step. This equals $D(2, 10) + 1 = 10 / \gcd(2, 10) + 1 = 6$, and corresponds to the degrees-of-freedom analysis in Section 5. The original notes thus independently arrived at the degrees-of-freedom count through a separate analytical path. The algorithm number for $N = 2$ in base 10 is 7, and the internal countdown follows Theorem 1 identically to all other cases.

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9. Conclusion

Every positive integer generates a perfect descending sequence within the modular frame of its successor. This is a consequence of the congruence $N \equiv -1 \pmod{N+1}$, and it means that every integer carries an intrinsic modular rhythm, revealed by reducing its multiplication products modulo its successor.

When this rhythm is observed within an external base, the difference between internal and external frame widths produces the algorithm number: a base-relative quantity specific to the entity-base pairing. A second base-relative quantity—the degrees of freedom—measures the breadth of the residue space accessible to the entity within the base. Together, these provide two distinct lenses on how a particular entity relates to a particular frame. Both are base-relative. Neither is preserved when the frame changes.

In every base, one entity achieves algorithm zero with full residue coverage: the base-complement, $N = B - 1$. This entity's internal frame width equals the external frame width. Its multiplication table visits every residue class. Among all bases in which it is frame-internal, it achieves these properties in exactly one.

We observe that the formal structure of this system—base-relative relational quantities, non-preservation across frames, and a unique entity with zero mismatch and full coverage—resembles properties discussed in theories of consciousness. We present this resemblance as analogy.

Algorithm Zero is the condition in which an entity's internal frame width equals the external frame width and its residue cycle covers every class in the base. This condition is achieved in one base among all admissible bases for any given entity. It is not preserved across bases. It arises from the specific coupling between a specific entity and a specific base.

Algorithms discovered by Dustin Sprenger, Formulation by Anthropic Claude, Cover image by Midjourney.

Entropy Breakpoint Hash – Algorithm Zero
(SHA-256 verified text. Reference UTF-8 source file for cryptographic validation)
427CD70A 140DFD58 ACD8743E 55422620 5D901CC6 D393E0D3 B2EE6222 CD0F58EF